EXERCISES

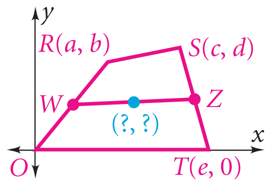
For more practice, see *Extra Practice.*

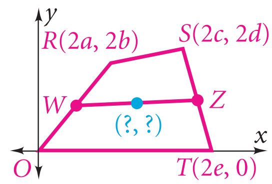
Practice and Problem Solving

A Practice by Example

Example 1
(page 332)

1. W and Z are the midpoints of $\overline{OR}$ and $\overline{ST}$, respectively. In parts (a)–(c), n d the coordinates of W and Z.

a.


b.


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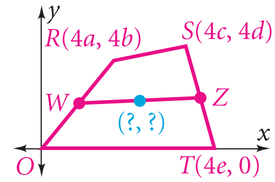


d. You are to plan a coordinate proof involving the midpoint of $\overline{WZ}$. Which of the figures (a)–(c) would you prefer to use? Explain.

Reading Math

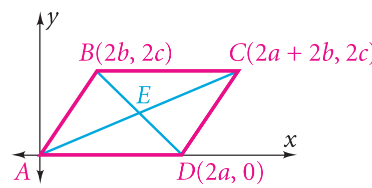
When you read large blocks of math text, cover all but a few lines to help you focus.

Developing Proof **Complete the plan for each coordinate proof.**

2. The diagonals of a parallelogram bisect each other (Theorem 6-3).

Given: Parallelogram $ABCD$

Prove: $\overline{AC}$ bisects $\overline{BD}$, and $\overline{BD}$ bisects $\overline{AC}$.

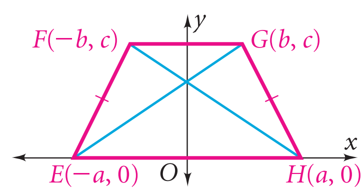


Plan: Place the parallelogram in the coordinate plane with a vertex at the **a.** __?__ and a base along the **b.** __?__. Since midpoints will be involved, use multiples of **c.** __?__ to name coordinates. To show segments bisect each other, show the midpoints have the same **d.** __?__.

3. The diagonals of an isosceles trapezoid are congruent (Theorem 6-16).

Given: Trapezoid $EFGH$ with $\overline{FE} \cong \overline{GH}$

Prove: $\overline{EG} \cong \overline{HF}$

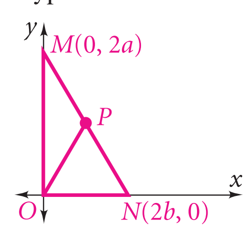


Plan: The trapezoid is isosceles, so place one base on the x-axis so that the **a.** __?__ bisects its bases. To show the diagonals are congruent, use the **b.** __?__ Formula.

4. The median to the hypotenuse of a right triangle is half the hypotenuse.

Given: $\triangle MNO$ is a right triangle with right $\angle MON$. P is the midpoint of $\overline{MN}$.

Prove: $OP = \frac{1}{2}MN$

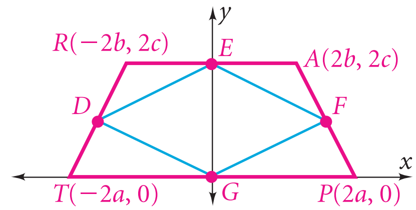


Plan: Place the right triangle in the coordinate plane with the vertex of the **a.** __?__ at the origin and the **b.** __?__ along each axis. Since midpoints will be involved, use **c.** __?__ to name coordinates for points **d.** __?__ and **e.** __?__. Use the **f.** __?__ Formula to find the coordinates of P. To compare lengths, use the **g.** __?__ Formula.

5. The segments joining the midpoints of consecutive sides of an isosceles trapezoid form a rhombus.

Given: Trapezoid $TRAP$ with $\overline{TR} \cong \overline{PA}$; D, E, F, and G are midpoints of the indicated sides.

Prove: $DEFG$ is a rhombus.



Plan: The trapezoid is **a.** __?__, so place one base on the **b.** __?__ so that the **c.** __?__ bisects its bases. Use multiples of 2 to name coordinates since **d.** __?__ will be involved. A rhombus is a parallelogram with four **e.** __?__ . To show opposite sides are parallel, show that their **f.** __?__ are the same. To show sides are congruent, use **g.** __?__.

Example 2 (page 333)

Developing Proof **Follow the plans above to complete the coordinate proofs.**

6. (Exercise 3) The diagonals of an isosceles trapezoid are congruent.

Proof: By the Distance Formula, EG = **a.** __?__ and HF = **b.** __?__. Therefore, $\overline{EG} \cong \overline{HF}$ by the de nition of congruence .

7. (Exercise 4) The median from the vertex of the right angle of a right triangle is half as long as the hypotenuse.

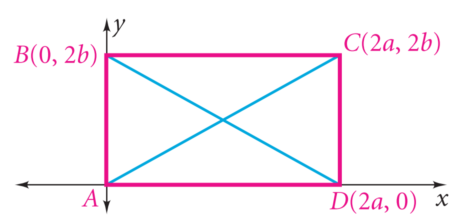
Proof: By the Distance Formula, OP = **a.** __?__ and MN = **b.** __?__. Therefore, $OP = \frac{1}{2}MN$.

8. (Exercise 5) The segments joining the midpoints of consecutive sides of an isosceles trapezoid form a rhombus.

Proof: The midpoints have coordinates **a.** D(__?__, __?__), E(__?__, __?__), F(__?__, __?__), and G(__?__, __?__). By the Distance Formula, DE = **b.** __?__, EF = **c.** __?__, FG = **d.** __?__, and GD = **e.** __?__. The slope of DE = **f.** __?__ and the slope of FG = **g.** __?__. The slope of EF = **h.** __?__ and that of GD = **i.** __?__. Thus, $DEFG$ is a parallelogram with congruent **j.** __?__, so **k.** __?__ is a rhombus by the definition of rhombus.

9. Use coordinate geometry to prove that the diagonals in the rectangular flag bisect each other (Theorem 6-3).

Proof: The midpoint of $\overline{AC}$ is **a.** __?__. The midpoint of $\overline{BD}$ is **b.** __?__. The midpoints are **c.** __?__, so the diagonals bisect each other.



B **Apply Your Skills**

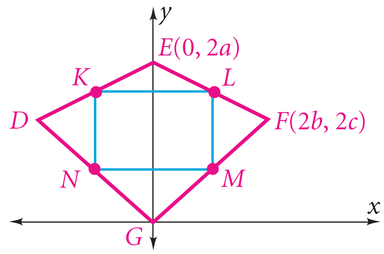
10. **Open-Ended** Give an example of a statement that you think is easier to prove with a coordinate geometry proof than with a paragraph, flow, or two-column proof. Explain your choice.

11. Developing Proof Complete the coordinate proof.

The midpoints of the sides of a kite determine a rectangle.

Given: Kite $DEFG$ with $DE = EF$ and $DG = GF$; K, L, M, and N are midpoints of the sides.

Prove: $KLMN$ is a rectangle.



Proof: The kite has two pairs of **a.** _?_ sides, so place congruent sides opposite each other across the y-axis. Use multiples of 2 to name coordinates since **b.** _?_ are involved. Name E as $(0, 2a)$. Name F as $(2b, 2c)$. Then, in terms of b and c, D must be **c.** _?_. By the Midpoint Formula, the midpoints are **d.** L(_?_, _?_), M(_?_, _?_), N(_?_, _?_), and K(_?_, _?_). Slopes for $\overline{KL}$ and $\overline{NM}$ are **e.** _?_, so $\overline{KL}$ and $\overline{NM}$ are parallel. Slopes for $\overline{KN}$ and $\overline{LM}$ are undefined, so $\overline{KN}$ and $\overline{LM}$ are **f.** _?_. Thus, opposite sides are **g.** _?_ and consecutive sides are **h.** _?_, so $KLMN$ is a rectangle.

Need Help?

Lines with unde ned slope are vertical lines. All vertical lines are parallel.

State whether each type of conclusion shown here could be reached using coordinate methods. Give a reason for each answer.

12. $\overline{AB} \cong \overline{CD}$

13. $\overline{AB} \parallel \overline{CD}$

14. $\overline{AB} \perp \overline{CD}$

15. $\overline{AB}$ bisects $\overline{CD}$.

16. $\overline{AB}$ bisects $\angle CAD$.

17. $\angle A \cong \angle B$

18. $\angle A$ is a right angle.

19. $AB + BC = AC$

20. $\triangle ABC$ is isosceles.

21. $\angle A$ and $\angle B$ are supplementary.

22. $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ are concurrent.

23. A, B, and C are collinear.

24. Quadrilateral $ABCD$ is a rhombus.

A and B have coordinates -2 and 10 on a number line. Find the coordinates of the points that separate $\overline{AB}$ into the given number of congruent segments.

25. 4 **26.** 6 **27.** 10 **28.** 50 **29.** n

The endpoints of $\overline{AB}$ are $A(-3, 5)$ and $B(9, 15)$. Find the coordinates of the points that separate $\overline{AB}$ into the given number of congruent segments.

30. 4 **31.** 6 **32.** 10 **33.** 50 **34.** n

Challenge

The endpoints of $\overline{AB}$ are as given. Find the coordinates of the points that separate $\overline{AB}$ into n congruent segments.

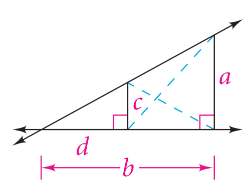
35. A has coordinate a and B has coordinate b on a number line.

36. A has coordinates (a, c) and B has coordinates (b, d) in the coordinate plane.

37. Use the diagram at the right.

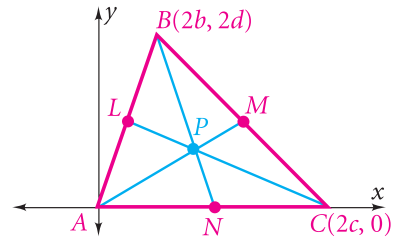
a. Explain using area why $\frac{1}{2}ad = \frac{1}{2}bc$ and hence $ad = bc$. (*Hint*: Area of triangle $= \frac{1}{2} \cdot$ base $\cdot$ height)

b. Use slope and part (a) to show: If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.



38. Physics For a mobile to be in balance, you suspend each part of the mobile at its center of mass. The center of mass, or centroid, is the point around which the weight of an object appears to be evenly distributed. You learned a method for finding the centroid of a triangle in Lesson 5-3. Now use it to help you find the centroid of a quadrilateral. (*Hint:* Where is the centroid of a segment?)

Proof 39. You learned in Lesson 5-3 (Theorem 5-8) that the centroid of a triangle, the point where the medians meet, is $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side. Complete the following steps to prove this theorem.



a. Find the coordinates of points L, M, and N, the midpoints of the sides of $\triangle ABC$.

b. Find equations of $\overleftrightarrow{AM}$, $\overleftrightarrow{BN}$, and $\overleftrightarrow{CL}$.

c. Find the coordinates of point P, the intersection of $\overleftrightarrow{AM}$ and $\overleftrightarrow{BN}$.

d. Show that point P is on $\overleftrightarrow{CL}$.

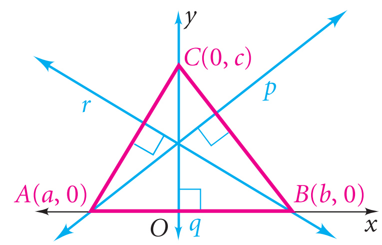
e. Use the Distance Formula to show that point P is $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side.

Need Help?

To show three lines are concurrent, you can show

1) one point is on all three lines
2) the intersection point of two lines is on the third line, or
3) the intersection of one pair of lines is the same as the intersection of another pair.

Proof 40. Complete the following steps to prove Theorem 5-9. You are given $\triangle ABC$ with altitudes p, q, and r. Show that p, q, and r intersect in a point (called the orthocenter of the triangle).



a. The slope of $\overline{BC}$ is $\frac{c}{-b}$. What is the slope of line p?

b. Show that the equation of line p is $y = \frac{b}{c}(x - a)$.

c. What is the equation of line q?

d. Show that lines p and q intersect at $\left(0, \frac{-ab}{c}\right)$.

e. The slope of $\overline{AC}$ is $\frac{c}{-a}$. What is the slope of line r?

f. Show that the equation of line r is $y = \frac{a}{c}(x - b)$.

g. Show that lines r and q intersect at $\left(0, \frac{-ab}{c}\right)$.

h. Give the coordinates of the orthocenter of $\triangle ABC$.

Proof 41. Write a coordinate proof of the theorem:

If the slopes of two lines have product -1, the lines are perpendicular.

a. First, argue that neither line can be horizontal or vertical.

b. Then, tell why the lines must intersect. (*Hint:* Use indirect reasoning.)

c. Knowing that they do intersect, place the lines in the coordinate plane, choose a point on ℓ_1, find a related point on ℓ_2, and complete the proof.

Standardized Test Prep

Multiple Choice

42. Two points on a line are $(-7, 10)$ and $(9, 2)$. Two points on a line parallel to that line are $(1, -3)$ and $(x, 4)$. What is the value of x?

A. -13 **B.** 13 **C.** 15 **D.** -15

43. Two points on a line are $(-4, 0)$ and $(8, 8)$. Two points on a line perpendicular to that line are $(8, -1)$ and $(6, y)$. What is the value of y?

F. 3 **G.** 2 **H.** $-\frac{7}{3}$ **I.** -4

Short Response

44. The endpoints of a segment are $(7, -3)$ and (a, b). The midpoint is $(3, 4)$.
- **a.** What are the coordinates of the other endpoint? Show your work.
- **b.** What is the length of the segment? Show your work.

Extended Response

45. Given: $\triangle ABC$; P, Q, and R are the midpoints of $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$, respectively.
- **a.** Place $\triangle ABC$ in the coordinate plane by writing coordinates for A, B, and C.
- **b.** What are the coordinates of P, Q, and R?
- **c.** Use coordinate geometry to prove $\triangle APQ \cong \triangle RQP$ by SSS.

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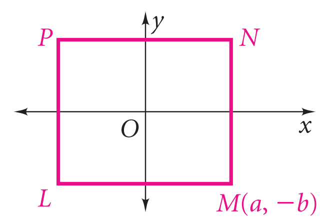
Mixed Review

Lesson 6-6

46. Rectangle $LMNP$ at the right is centered at the origin. Give coordinates for point P without using any new variables.



Lesson 5-4

Write (a) the inverse and (b) the contrapositive of each statement.

47. If the sum of the angles of a polygon is not 360°, then the polygon is not a quadrilateral.

48. If $x = 51$, then $2x = 102$.

49. If $a = 5$, then $a^2 = 25$.

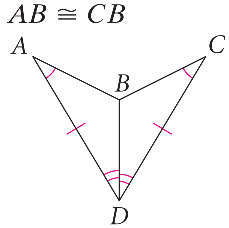
50. If $b < -4$, then b is negative.

51. If $c > 0$, then c is positive.

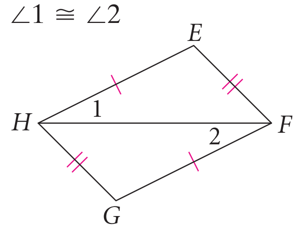
Lesson 4-4

Explain how you can use SSS, SAS, ASA, or AAS with CPCTC to prove each statement true.

52. $\overline{AB} \cong \overline{CB}$



53. $\angle 1 \cong \angle 2$



54. $\angle K \cong \angle M$